

# Dark Energy as a Born-Infeld Gauge Interaction Violating the Equivalence Principle

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We investigate the possibility that dark energy does not couple to gravitation in the same way as ordinary matter, yielding a violation of the weak and strong equivalence principles on cosmological scales. We build a transient mechanism in which gravitation is pushed away from general relativity by a Born-Infeld gauge interaction acting as an *Abnormally Weighting (dark) Energy (AWE)*. This mechanism accounts for the Hubble diagram of far-away supernovae by cosmic acceleration and time variation of the gravitational constant while accounting naturally for the present tests on general relativity.

To account for the dimmed magnitude of type Ia supernovae (see [1] and references therein), it is necessary to invoke a recent acceleration of the cosmic expansion, provided these objects can be considered as standard candles. This usual explanation does not require to give up general relativity (GR) as it includes naturally a way to accelerate cosmic expansion through a positive cosmological constant  $\Lambda$ . In the standard cosmological picture, based on GR, gravitation contains only spin 2 gravitational degrees of freedom (the metric field  $g_{\mu\nu}$ ) and obeys the equivalence principle. Under the assumptions of the cosmological principle, the corresponding geometry for space-time is locally given by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element:

$$ds^2 = -dt^2 + a^2(t) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2) \quad (1)$$

where  $a(t)$  is the scale factor and where we assumed synchronous time coordinate<sup>1</sup>. In this framework, the cosmic acceleration is ruled by the following equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G (\rho + 3p), \quad (2)$$

where  $\rho$  and  $p$  stand for the energy density and pressure of the matter filling space-time. In order to provide the necessary cosmic acceleration ( $\ddot{a} > 0$ ), it is therefore compulsory to violate the strong energy condition (SEC) [2]:  $p < -\rho/3$ . The Einstein cosmological constant  $\Lambda$  is the usual way used to provide cosmic acceleration, although it leads to the intricate problems of fine-tuning ( $\rho_\Lambda^{th} \approx m_{Pl}^4 \approx 10^{76} GeV^4$ ) and coincidence ( $\rho_\Lambda^{obs} \approx \rho_{c,0} = 3H_0^2/(8\pi G) \approx 10^{-47} GeV^4$ ) once  $\Lambda$  is interpreted as non-vanishing vacuum energy density (cf. [3] for a review and [4] for an interesting alternate interpretation). Most of the alternate explanations, like quintessence, also require to violate the SEC.

In this letter, we propose a completely new interpretation of dark energy that does not require this violation. Instead, we assume that "dark" energy violates the weak equivalence principle (WEP) on large-scales, i.e. it does not couple to gravitation as usual matter and weights abnormally. Doing so, its related gravitational binding energy will be felt differently by other types of matter, therefore violating also the strong equivalence principle (SEP). Under these assumptions, we build a dark energy mechanism without violation of the SEC. This *Abnormally Weighting Energy (AWE)* will consist here of an additional gauge interaction of Born-Infeld (BI) type which will provide a natural scheme for transient dark energy mechanism. This will lead to a satisfactory explanation of Hubble diagrams of type Ia supernovae while still accounting for the stringent constraints on GR we know today.

Here, we will consider that the energy content of the universe is divided into three parts : a gravitational sector described by pure spin 2 (graviton) and spin 0 (dilaton) degrees of freedom, a matter sector containing the usual fluids of cosmology (baryons, photons, dark matter, ...) and an AWE sector, here composed by a gauge interaction ruled by BI type gauge dynamics. The introduction of a scalar partner to the graviton is necessary to account for the violation of the equivalence principle. The violation of the WEP by the AWE can be represented by different couplings between gravity, the AWE and usual matter:

$$S = \frac{1}{2\kappa} \int \sqrt{-g} d^4x \{ R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \} \\ + S_{BI} [A_\mu, A_{BI}^2(\varphi) g_{\mu\nu}] \\ + S_m [\psi_m, A_m^2(\varphi) g_{\mu\nu}], \quad (3)$$

where  $\kappa$  is the "bare" gravitational coupling constant. In the previous action,  $g_{\mu\nu}$  is the Einstein metric,  $\varphi$  is a gravitational scalar field,  $S_{BI}$  is a gauge AWE sector ruled by BI dynamics and  $S_m$  is the usual matter sector with matter fields  $\psi_m$ ;  $A_{BI}(\varphi)$  and  $A_m(\varphi)$  being the coupling functions to the metric  $g_{\mu\nu}$  for the AWE and matter sectors respectively. The non-universality of the gravitational couplings ( $A_{BI} \neq A_m$ ) yields a violation of

<sup>1</sup> Here, we have also restricted ourselves to the case of flat space-times for the sake of simplicity. Throughout this paper, we will assume the Planck system of units, in which  $\hbar = c = 1$ ,  $G = m_{Pl}^{-2}$  and the gravitational coupling constant is  $\kappa = 8\pi G$ .

the WEP: experiments using the new BI gauge interaction would provide a different inertial mass than all other experiments<sup>2</sup>. The action (3) is written in the so-called "*Einstein frame*" where the metric components are measured by using purely gravitational rods and clocks, i.e. not build upon any of the matter fields nor the ones from the AWE sector. We will define the "*Dicke-Jordan*" observable frame by the conformal transformation

$$\tilde{g}_{\mu\nu} = A_m^2(\varphi)g_{\mu\nu} \quad (4)$$

using the coupling function to ordinary matter. Indeed, in this frame, the metric  $\tilde{g}_{\mu\nu}$  couples universally to ordinary matter and is measured by clocks and rods made of usual matter (but not build upon the new gauge interaction we introduced as the AWE sector). The violation of the WEP therefore only concerns the new gauge sector that was introduced in (3). Throughout this paper, quantities with a tilde will refer to the observable frame given by (4).

BI gauge dynamics allows to avoid point-like singularities in the field strength through classical vacuum polarization effects by freezing the gauge potentials above some given critical energy  $\epsilon_c$ . This can be done by assuming the lagrangian  $\mathcal{L}_{BI} = \epsilon_c(\mathcal{R} - 1)$  for the gauge field, where

$$\mathcal{R} = \sqrt{1 + A_{BI}^{-4}/(2\epsilon_c) F_{\mu\nu}F^{\mu\nu} - A_{BI}^{-8}/(16\epsilon_c^2) (F_{\mu\nu}\tilde{F}^{\mu\nu})^2}$$

(see [6] and references therein). At low-energies, the BI gauge dynamics reduces to Yang-Mills dynamics where the gauge fields are radiative. In a cosmological context (see [6] for a complete study of cosmology with BI gauge fields and [7] for the introduction of the dilaton  $\varphi$  in the model), such BI gauge fields obey the following equation of state:

$$\omega_{BI} = \frac{p_{BI}}{\rho_{BI}} = \frac{1}{3} \left( \frac{\epsilon_c - A_{BI}^{-4}(\varphi)\rho_{BI}}{\epsilon_c + A_{BI}^{-4}(\varphi)\rho_{BI}} \right), \quad (5)$$

where  $\rho_{BI}$  ( $p_{BI}$ ) is the gauge energy density (pressure) in the Einstein frame. As the only coupling between AWE and matter is purely gravitational, the scaling evolution of the gauge energy density is

$$\rho_{BI} = \epsilon_c A_{BI}^4(\varphi) \left( \sqrt{1 + \mathcal{C}/(A_{BI}^4(\varphi)a^4)} - 1 \right)$$

( $\mathcal{C}$  is an integration constant). When the condition  $A_{BI}^{-4}(\varphi)\rho_{BI} \gg \epsilon_c$  occurs, the gauge field pressure is negative  $p_{BI}/\rho_{BI} \approx -1/3$ , and the related gauge field energy density scales as  $(A_{BI}(\varphi)a)^{-2}$  (frozen field strength).

However, in the low-energy regime

$A_{BI}^{-4}(\varphi)\rho_{BI} \ll \epsilon_c$ , the fluid becomes relativistic  $p_{BI}/\rho_{BI} \approx 1/3$ . This remarkable equation of state allows a possible transient domination of the BI energy.

The general cosmological dynamics of the action (3) have been studied in details in [7] for various couplings of the gauge field to the dilaton, while the case of BI gauge fields alone can be found already in [6]. Here, we will focus on describing the transient dark energy mechanism based on this dynamics. The Friedmann equation obtained from the action (3) writes down

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{\dot{\varphi}^2}{3} + \frac{\kappa}{3} [\rho_{BI} + \rho_m], \quad (6)$$

where a dot denotes a derivative with respect to the time coordinate  $t$ , and  $\rho_m$  is the energy density of the matter sector (Einstein frame). The acceleration equation is given by

$$\frac{\ddot{a}}{a} = -\frac{2}{3}\dot{\varphi}^2 - \frac{\kappa}{6} [(\rho_{BI} + 3p_{BI}) + (\rho_m + 3p_m)]. \quad (7)$$

There cannot be any cosmic acceleration in terms of the metric  $g_{\mu\nu}$  (the dilaton  $\varphi$  is here massless), as the highest value of  $\ddot{a}$  that can be achieved in this frame is identically zero from (7) (see [7]), as the BI gauge interaction never violates the SEC. As there is no direct coupling between the gauge and the matter sectors in (3), the behavior of the matter energy density and pressure are given as in usual tensor-scalar gravity. These quantities are given in the observable frame (4) by  $\tilde{\rho}_m = A_m^{-4}(\varphi)\rho_m$  where  $\rho_m$  represents these quantities expressed in the Einstein frame (with similar relation for the pressure). In this frame, they have the same scaling law as in standard cosmology based on GR.

The scalar gravitational dynamics are given by the Klein-Gordon equation:

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + \frac{\kappa}{2}\alpha_{BI}(\varphi)(\rho_{BI} - 3p_{BI}) + \frac{\kappa}{2}\alpha_m(\varphi)(\rho_m - 3p_m) = 0, \quad (8)$$

where  $\alpha_i(\varphi) = d \ln A_i(\varphi)/d\varphi$ . The violation of the WEP induced by the AWE sector ( $\alpha_{BI} \neq \alpha_m$ ) implies that the history of the universe can be seen as a competition between usual matter and AWE, particularly if the first attracts the field toward values corresponding to GR (here  $\varphi = 0$  and  $\dot{\varphi} = 0$ ) while the last acts as a repulsion from it. As the AWE sector is here constituted by a BI gauge interaction, this competition will be temporary because of the equation of state (5). At high-energies, the negative pressure will first allow a late domination of AWE, while at low-energies the radiation behavior ( $\omega_{BI} = 1/3$ ) will ensure both a decoupling of the AWE sector from the scalar field (see (8)) and a final sub-dominance of AWE. The resulting dark energy mechanism is therefore transient. A well-known and remarkable feature of

<sup>2</sup> Furthermore, non-universal couplings to the dilaton arise naturally in string theory, see [5] for example.

tensor-scalar theories of gravitation is their convergence towards general relativity during matter-dominated era (see [8] and references therein), which is ensured when the coupling function  $\alpha_m(\varphi)$  has a global minimum or which can be achieved provided specific initial conditions for general coupling functions. In order to introduce a competition between attraction by ordinary matter and repulsion by AWE in (8), it suffices to assume the usual coupling functions:  $A_{BI}(\varphi) = \exp(k_{BI}\varphi)$  and  $A_m(\varphi) = \exp\left(k_m \frac{\varphi^2}{2}\right)$ . The deviation from GR occurs when the dilaton  $\varphi$  is pushed away from the minimum of the matter coupling function  $A_m(\varphi)$  (GR) by the constant drag term ( $\alpha_{BI} = k_{BI}$ ) when the BI term in (8),  $\alpha_{BI}(\varphi)(\rho_{BI} - 3p_{BI}) \gg \alpha_m(\varphi)\rho_m$ , dominates the scalar dynamics (matter-dominated era  $p_m \approx 0$ ). However, convergence to GR is ensured by the efficiency of the attraction mechanism associated to the coupling function  $\alpha_m = k_m\varphi$ , provided the matter force term in (8) dominates, which occurs twice. The first time is before the AWE dominance, when the BI gauge interaction was subdominant, and this allows to account for the validity of GR in the early times of Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN). The second time is at the end of the process as soon as the BI gauge interaction behaves like radiation, i.e. even if it is still dominating the energy content of the universe.

Let us now illustrate this mechanism, where dark AWE never violates the SEC  $p > -\rho/3$  in the Einstein frame, by reproducing a Hubble diagram built upon recent available data on far-away type Ia supernovae [1]. Within the framework of tensor-scalar gravity, the dimmed magnitude of such objects could be explained both by an acceleration of cosmic expansion and a time variation of the gravitational constant. In [9, 10], the following toy model for the moduli distance vs redshift relation of type Ia supernovae has been proposed:

$$\mu(\tilde{z}) = m - M = 5 \log_{10} d_L(\tilde{z}) + \frac{15}{4} \log_{10} \frac{G_{eff}(\tilde{z})}{G_0}, \quad (9)$$

where  $d_L(\tilde{z})$  is the luminous distance (in Mpc) given by  $d_L(\tilde{z}) = (1 + \tilde{z})\tilde{H}_0 \int_0^{\tilde{z}} d\tilde{z}/\tilde{H}(\tilde{z})$  for a flat universe ( $\tilde{H}_0$  is the observed value of the Hubble constant today). The expansion rate  $\tilde{H}(\tilde{z})$  has to be estimated in the observable frame related to usual matter (4) ( $\tilde{H} = d\tilde{a}/(\tilde{a}d\tilde{t}) = A_m^{-1}(\varphi)(H + \alpha_m(\varphi)\dot{\varphi})$ , with  $H = \dot{a}/a$  is the Hubble parameter in the Einstein frame). In (9),  $G_{eff}$  is the effective gravitational "constant" at the epoch  $\tilde{z}$ :

$$G_N = G_0 A_m^2(\varphi)(1 + \alpha_m^2(\varphi)). \quad (10)$$

where  $G_0$  is the (bare) value of this constant today, where gravitation is well-described by GR. In addition to account for moduli distance data, any dark energy mechanism based on tensor-scalar theory of gravitation should be in agreement with the present tests of GR, which concern only usual matter and not AWE. The constraints on

post-newtonian parameters are given by (cf. [11, 12]):

$$|\gamma - 1| = 2 \frac{\alpha_m^2(\varphi)}{(1 + \alpha_m^2(\varphi))} < 2 \times 10^{-5} \quad (11)$$

$$|\beta - 1| = \left| \frac{d\alpha_m}{d\varphi} \frac{\alpha_m^2(\varphi)}{2(1 + \alpha_m^2(\varphi))} \right| < 6 \times 10^{-4}. \quad (12)$$

Another constraint is the time-variation of the gravitational constant [13]:

$$\left| \frac{\dot{G}}{G} \right| = \left| 2\dot{\varphi}\alpha_m(\varphi) \frac{1 + \frac{d\alpha_m}{d\varphi}}{1 + \alpha_m^2(\varphi)} \right| < 6 \times 10^{-12} yr^{-1}. \quad (13)$$

One should also add the constraints on the WEP, which is tested at the  $10^{-12}$  level by the universality of free fall of inertial masses with different compositions [14]. Although the AWE violates this universality of free fall, we might consider that this effect is extremely weak (and not observed in practice) provided the AWE density at our scale is of the order of its cosmological value ( $\rho_{BI,0} \approx \rho_{c,0}$ ). This is true if the AWE does not cluster too much at our scale, an assumption that should be verified in forthcoming works. Therefore, we will only consider the constraints (11), (12) and (13) while discarding the effects on the universality of free fall for the moment.

The dark energy model proposed here actually depends on four free parameters: the initial BI energy density  $\tilde{\rho}_{BI}(a_i)$ , the critical BI energy  $\epsilon_c$ , the parameters  $k_{BI}$  and  $k_m$  of the two dilaton couplings to the AWE and to matter, respectively. Once the cosmological evolution is determined (see [7] for details), the observable quantities are derived using (4) and (10). The parameters  $\tilde{\rho}_{BI}(a_i)$  and  $\epsilon_c$  are chosen such that  $\tilde{\Omega}_m(a_0) \approx 0.3$  (flat universe), the observable energy contributions being given by performing the conformal transformation (4) on the Friedmann equation (6). Figure 1 illustrates the adequacy of the model to a Hubble diagram of type Ia supernovae. As a matter of comparison, we also give the value of the  $\chi$ -square, marginalised over  $H_0$ , per degrees of freedom denoted by  $\bar{\chi}^2/dof$ . The model was first set by minimising the  $\bar{\chi}^2$  to a value very close to the best fit  $\Lambda$ CDM model. Then, as the constraints (11), (12) and (13) were not completely satisfied for these best fit models, we pushed the time-integration a little bit further to let the attraction mechanism fix this naturally. This resulted in a slightly increased value of  $\bar{\chi}^2/dof$ . We will not go deeper here into these statistical issues, as our aim is only to illustrate the dark energy mechanism described here.

Figure 2a) represents the corresponding cosmological evolution of the effective gravitational constant (10) while Figure 2b) illustrates the cosmic acceleration. For the correction due to a variable  $G_N$  in the toy model (9), the model does not require any cosmic acceleration (see Figure 2b)). However, we have found that if the correction

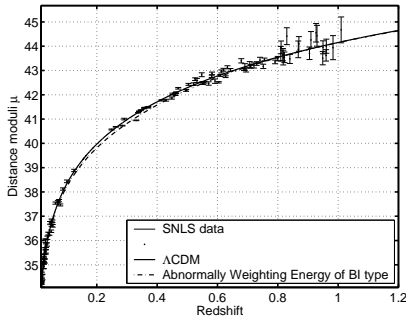


FIG. 1: Hubble diagram of SNLS 1st year data set with the best fit  $\Lambda$ CDM flat model (solid line,  $\Omega_m(a_0) = 0.26$ ,  $\bar{\chi}^2/dof = 1.03$ ) and the AWE model (dash-dotted line,  $\bar{\chi}^2/dof = 1.09$ ) ( $H_0 = 70 \text{ km/s/Mpc}$ )

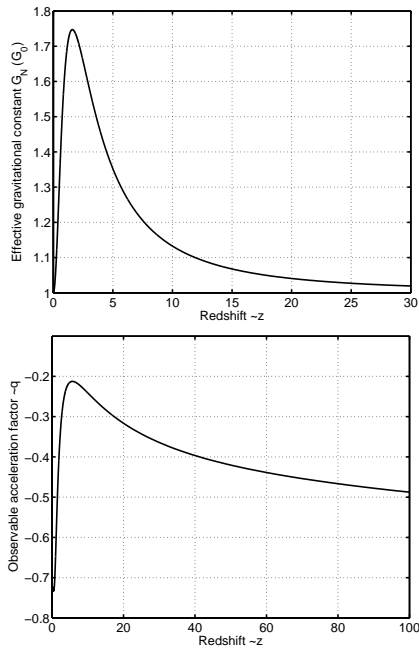


FIG. 2: a) Cosmological evolution of  $G_N$  (in units of bare  $G$ ) b) Evolution of the acceleration factor  $\tilde{q} = 1 + (d\tilde{H}/d\tilde{t}) / (\tilde{H}^2)$  with the redshift  $\tilde{z}$  in the observable frame

in  $G_N$  has been over-estimated in (9), transient cosmic acceleration is needed to match data. This acceleration is only due to the interpretation in the observable frame (4) and not to a violation of the SEC in the Einstein frame (see also [7]). Therefore, dark energy effects consist here of a combination of variable  $G_N$  and transient cosmic acceleration. Figure 3 represents the evolution of the post-newtonian parameters (11), (12) and the constraints on the absolute variation of  $G_N$  (13). The convergence occurs during domination of the AWE sector because of the decoupling of the BI gauge interaction once it reaches its radiative regime. The history of the mechanism is as follows : the BI gauge field starts sub-dominant at the end

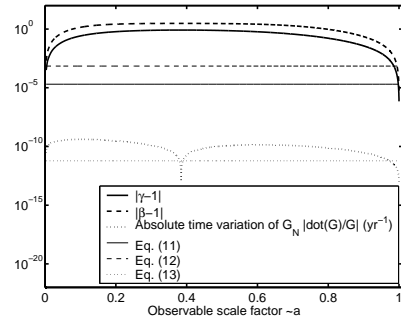


FIG. 3: Evolution of post-newtonian parameters with the scale factor  $|\gamma - 1|$  (solid line); b)  $|\beta - 1|$  (dashed line) ; c)  $|\dot{G}/G|$  (dots). Current observable constraints are indicated by the horizontal lines

of the radiation-dominated era while gravitation is well described by GR. Then, as the energy densities progressively cool down to coincidence the scalar field is pushed away from GR by the increasing repulsive influence of the AWE. This repulsive influence rapidly decreases as the gauge field becomes radiative and decouples from the scalar sector. Between this period and today, matter becomes the dominant driving term and attracts towards GR to finally achieve the level of precision we know for it today. However, during a short period of time in the very recent cosmic history, gravitation was substantially different from GR and led to dark energy effects.

This letter has presented a new dark energy mechanism where this energy weighs abnormally. This violates the WEP which obviously leads to a violation of the SEP as modeled by a tensor-scalar theory of gravitation. As a consequence, the "dark" AWE does not need anymore to exert too negative pressures (and a violation of the SEC) to achieve its job efficiently. The BI gauge interaction used as AWE provides here a natural transient dark energy mechanism compatible with supernovae data, constraints on GR today and during the radiative era. However, this mechanism is likely to have a strong impact on physics in the matter-dominated era by the variation of  $G_N$  and the acceleration it yields. As well, the clustering of such AWE should also lead to a violation of the universality of free fall. A careful study of all these effects could therefore determine whether some processes in the universe are not ruled by the equivalence principle. If proved true, this would completely change our views of gravitation and the universe.

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